Performance Modeling of Secondary Users in CRNs with Heterogeneous Channels

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***Abstract*—The goal of this paper is to model heterogeneous channel Access in Cognitive Radio Networks (CRNs). In CRNs, when licensed users, known as Primary Users (PUs), are idle, unlicensed users, known as Secondary Users (SUs) can use their assigned channels. In the model we consider in this paper, there are two types of licensed channels, where one type has a larger bandwidth, and hence a higher service rate for SUs. Therefore, SUs prefer to use such channels, if available, over channels in the second type which have a lower service rate. SUs may also switch from the second to the first type of channels when they become available, even if their current channels are still available. Moreover in our performance model, we model the SUs' sensing process, and its dependence on the system load, and number of sensing users. We use a Continuous Time Markov Chain ($CTMC$) modeling approach, and derive SUs' performance metrics, which include SUs admission and blocking probabilities, and their average waiting time in the system. We also develop a baseline model in which SUs do not switch channels between the two types, unless they are interrupted by PUs, and compare its performance to our proposed model. Our numerical analysis shows that our proposed model outperforms the baseline model. We also, found that if sensing time is very small (≤ 1 ms), its effect on SUs performance is insignificant.**

I. Introduction

A. Background

Due to the temporal and spatial underutilization of licensed spectrum bands, as well as the crowdedness of unlicensed bands, a new spectrum access paradigm has been recently proposed namely, Cognitive Radio (CR) [1]. CR enables users to adjust their transceivers' frequencies depending on the availability of licensed frequency bands which are otherwise unused by their licensees [2]. Thus, unlicensed wireless users, called Secondary Users (SUs) can dynamically and opportunistically access unused licensed bands in order to improve their throughput and service reliability. In this case, whenever the licensed or the Primary Users (PUs) become active, SUs must vacate their bands.

CRNs have many challenges such as spectrum sensing, management, mobility, allocation and sharing [3], [4]. Usually, SUs have QoS performance requirements, e.g., throughput and maximum transmission delay. Evaluating these metrics is not a trivial task, due to the CRNs dynamic nature, e.g., due to PUs fluctuating activities which may interrupt SUs, and hence may need to access the channel multiple times just to finish one communication session. To evaluate these performance metrics, a few models have been proposed in literature. In [5], [6], a Markovian model is proposed to analyze spectrum access with and without buffering for new and interrupted SUs requests, which is used to evaluate SUs mean waiting time, and the probabilities of blocking, interruption, forced termination, and non-completion. Results show that buffering SUs requests reduces SUs' blocking and non-completion probabilities, with a very small increase of forced termination probability. However, **in all other models** network channels are assumed symmetric in terms of channels bandwidth. In addition, SUs' sensing overhead is not considered in those models.

A quasi-birth and death Markov chain with continuous time and state space model is proposed in [7], to improve SUs performance by distributing their flows to multiple wireless networks. Due to the high complexity for this model, an approximation solution was proposed. They proposed two admission control schemes for SU flows priority, and no priority schemes. In both admission control schemes, if an SU is admitted to a network, it will not leave it until finishing its transmission as long as it is not interrupted by a PU arrival.

A Markov model for spectrum sharing between PUs and SUs is proposed in [8], when SUs are interrupted, they are suspended and wait to access another channel in a call level queue. During SUs' suspension, packets generated by SUs are either delayed or discarded, therefore the queue becomes two sub-queues, delay and discard queues. Three metrics are evaluated: packet loss ratio, packet delay, and throughput. Results show by increasing SU suspension queue length, both packet loss ratio and throughput increase and packet delay decreases. A queuing network model for spectrum sharing between PUs and SUs is introduced in [9], where a closed form solution for equilibrium system state was derived as a generating function. The model studies PUs QoS degradation due to unreliable SUs spectrum sensing where an SU keeps on using the channel, although a PU has arrived to the same channel. Besides, an SU moves from its channel to another only when it is interrupted by a PU arrival. Channels are assumed symmetric in terms of service rate for PUs and SUs in [8], [9].

B. Motivation

This work is motivated by:

- **First**, the fact that heterogeneous channels may be present in the same locality, e.g., TV channels, cellular telephone channels, wireless microphone channels, etc, which might be used by SUs, if available. The bandwidth availability in these channels is different. For instance, the licensed spectrum of Wireless Microphones has 200 KHz bandwidth, and that of Digital TV (ATSC) has 6 MHz bandwidth. Therefore, this work is motivated by the possibility

of SUs switching channels opportunistically in order to improve their performance.
- **Second**, the fact that some of the PUs of those channels characterized by long idle times, e.g., Digital TV channels, which may lead to a sustainable SU throughput, which also reduces channels switching overhead.
- **Third**, this work is also motivated by the need to consider sensing time and its effect on channel utilization and transmission delay. There are different technologies for spectrum sensing such as energy and feature detection. Energy detection sensing is frequent, and its typical sensing time is less than 1 ms, while feature detection, such as cyclostationary detection, is less frequent and sensing time is around 24.2 ms for Digital TV [13].

We are therefore motivated to develop a modeling approach that considers these three important issues and allows one to evaluate the performance of SUs under these realistic conditions.

C. Paper Contributions

The contributions of this paper are as follows:

1) We introduce a performance model for CRNs that models heterogeneous channels, as well as the sensing process in a manner that is dependent on the load. That is, the sensing time increases if fewer channels become available, and if fewer SUs are available to sense channels.
2) We introduce a strategy that gives preference to access channels with potentially larger idle times and higher bandwidth.
3) Through numerical results, we show that our proposed strategy outperforms a baseline model that does not allow switching between channels. In particular, our proposed strategy reduces the mean waiting time for SUs in the system.
4) Also, our numerical results show if sensing time is very small ($\leq$ 1 ms), its effect on SUs performance is insignificant.

D. Paper Organization

The rest of this paper is organized as follows. The model and assumptions are explained in Section II. In Section III, our proposed Continuous Time Markov Chains ($CTMC$) model is presented. Performance metrics are derived in Section IV. The baseline model is described in Section V, which is used for comparison to our proposed model. Numerical results and discussions are presented in Section VI. We conclude the paper in Section VII.

II. Model and Assumptions

We use a mixed queuing network to model the CRN system where PUs are modeled as a closed chain of customers, while SUs are modeled as an open chain. Table I shows the notation and their description. Our proposed model contains two types of channels, C_1 and C_2, and a Virtual Queue (VQ), which is used to accommodate SUs. It is assumed that channels in C_1 have a higher bandwidth than channels in C_2, and SUs, therefore, prefer to use channels in C_1. SUs may move from the VQ to a channel in C_1, if available, as their first preference.

TABLE I
Table of Notations, where $i = \{1, 2\}$.

Notation	Description
C_i	Number of channels in type i.
v	Number of SUs in the system: VQ, C1, and C2.
β	The maximum size of the VQ buffer.
s	A binary variable for sensing state, where 0 means no sensing is being conducted. 1, otherwise.
p_i	Number of busy PUs in type i channels.
η	SUs sensing rate.
$\Psi_{(p_1+p_2,v)}$	SUs sensing rate function.
p_f	Probability of false alarm.
λ_s	SUs arrival rate.
λ_{p_i}	PUs arrival rate in type i channels.
μ_{s_i}	SUs service rate in type i channels.
μ_{p_i}	PUs service rate in type i channels.
$\{v, p_1, p_2, s\}$	A system state where v, p_1, p_2, and s are the number of SUs in system, busy PUs in C_1, busy PUs in C_2, and sensing state.
$\pi_{v,p_1,p_2,s}$	The probability of a steady state $\{v, p_1, p_2, s\}$.
p_b	Probability of SUs blocking, for C_1 and C_2 overall.
p_a	Probability of SUs admission, for C_1 and C_2 overall.
$\overline{L}$	Average number of SUs in the system: VQ, C_1, and C_2.
$\overline{W}$	SUs average waiting time in the system: VQ, C_1, and C_2, until finishing their packet transmission.

Otherwise, they move to a channel in C_2, if available. In this model, whenever an SU detects an available channel in C_1, the SU starts using this channel, although the SU may have been using a channel in C_2. The purpose for doing so is to improve the SUs performance. SUs detect out-of-band channels availability by exchanging control information over Common Control Channel (CCC) [10]. The maximum number of SUs which can be in the system equals the VQ buffer size, β. Therefore, if an SU is interrupted during its service by the channel's PU, the SU moves to the VQ, and starts to sense for available channels in order to finish its own transmission, and then leaves the system.

A. Assumptions

- SUs exchange channels' information, such as channel's availability using a CCC [11], which is assumed to be always available.
- Each channel has its own PU assigned to it.
- The number of SUs in the network is unlimited, but a maximum of β can be in the system, either occupying channels, or waiting for channels to become available.
- Each channel is modeled as a server with no buffer.
- There are two types of channels, type 1 and type 2.

- ◦ SUs assign a higher priority for using type 1 channels over type 2, due to the higher throughput of type 1 channels.
- ◦ Assume each SU has two transceivers, one for data transmission, and the other for exchanging control packets with other SUs over the CCC, and for conducting in band and out-of-band sensing.
- ◦ The Virtual Queue (VQ) is a concept to hold the newly arrived and interrupted SUs, as well as SUs being served.
- ◦ When an SU finishes its transmission, the SU leaves the network. However, if an SU is interrupted, the SU moves to the VQ and waits for an available channel in order to complete its transmission.
- ◦ When a PU or an SU finishes its transmission on a type 1 channel, e.g., channel k, then an SU being served on a type 2 channel, if any, moves to channel k, in order to improve the SUs throughput.
- ◦ SUs sensing time[1] is exponentially distributed with a rate that is dependent on v, p_1, and p_2. Let $\Psi_{(p_1+p_2,v)}$ be this sensing rate function, and it will be defined in the numerical results Section, Section VI, equation (13). Practically, $\Psi_{(p_1+p_2,v)} >> \lambda_s$, λ_{p_1}, λ_{p_2}, μ_{s_1}, μ_{s_2}, μ_{p_1}, and μ_{p_2}.
- ◦ Sensing is triggered when an SU arrives, given there is an idle channel. Or, when a PU or an SU finishes transmission and there is at least one SU waiting in the VQ.
- ◦ We only model good sensing which results in finding an idle channel that the SU can use. Modeling sensing which does not result in accessing a channel, either because all channels are busy, or because there are no waiting SUs, will have no bearing on the system operation, and does not change the model.
- ◦ The probability of misdetection under sensing is assumed to be very small, and is therefore negligible, in order to reduce the model complexity. Misdetection is defined as detecting the channel as idle, while the channel is occupied by a PU's transmission.
- ◦ The probability of false alarm, p_f, is considered in this model, since it has much higher effect than the probability of misdetection in our system model, and it is usually less than 0.1, e.g., as in IEEE 802.22 CRNs standard [12]. False alarm is defined as detecting the channel as busy by a PU's transmission, while in reality it is idle.

B. Parameters

- ◦ C_1 and C_2 are the number of channels of types 1 and 2, respectively, and are also the number of PUs assigned to these channels.
- ◦ PUs assigned to type 1 and type 2 channels, have exponentially distributed inter-arrival times with rates of λ_{p_1} and λ_{p_2}, respectively, when they are idle.
- ◦ PUs using type 1 and type 2 channels have service rates of μ_{p_1} and μ_{p_2}, respectively, with exponential distributions, when they are active.
- ◦ β is the maximum size of the VQ[2].

[1] In this paper, it is assumed that what we refer to as the sensing time, includes both the channel sensing, and channel switching times.

[2] In our numerical results, β is set to a large value such that SUs $p_a \approx 1$.

- ◦ SUs arrival rates to the VQ is λ_s with Poisson distributions.
- ◦ On type 1 and type 2 channels, the SUs service rates are μ_{s_1} and μ_{s_2}, respectively, with an exponential distributions.

C. Variables

- ◦ s is a binary variable, {0,1}, for sensing state, where 0 means no sensing is being conducted. While 1, otherwise.
- ◦ v is the total number of SUs in the VQ, including those being served by types 1 and 2 channels, while in this model v is taken as finite, the virtual queue size, β, can also be set to a very large number, hence approximating the infinite number of SUs case, as will be shown in Section VI.
- ◦ p_1 and p_2 are the numbers of PUs being served by type 1 and type 2 channels, respectively.
- ◦ For our model, we define the state space, call it ζ, as (v, p_1, p_2, s), where: $0 \leq v \leq \beta$, $0 \leq p_1 \leq C_1$, $0 \leq p_2 \leq C_2$, and s is a binary variable, such that v must be ≥ 1, if $s = 1$, which means at least one SU must exist to conduct sensing.
- ◦ let $\pi_{\hat{v},\hat{p_1},\hat{p_2},s}$ be the stationary probability vector where $v = \hat{v}$, $p_1 = \hat{p_1}$, $p_2 = \hat{p_2}$, and s is a binary variable such that if $s = 1$, sensing is being conducted by SU(s). Otherwise; no sensing.

It is to be noted that the assumption of memoryless distributions, i.e., exponential distributions, has been made in order to make the model mathematically tractable. This is a standard assumption that is made in such complicated models. It is to be also noted that the queuing network is ergodic, because it is irreducible and has a finite state space. The queueing network therefore has a unique steady state (S.S.) distribution, $\overrightarrow{\pi}$.

III. Model Formulation

There are 5 cases of the global balance equations. Since sensing is inconsequential when no channels are available, it is assumed that sensing is terminated when a PU arrives to occupy its channel and no other channels are available. It is also assumed that at most one decision can be made based on sensing at the same time.

In order to model the system exactly, a greater number of state variables need to be included, which will significantly increase the system complexity. Therefore, we introduce two relaxations which result on bounds on system performance. These are an optimistic bound and a pessimistic bound. The definition of these two models are as follows:

- For the **optimistic bound analysis**, if sensing is conducted, then we assume it is on a channel in type 2 (lower SUs service rate), i.e., all available channels in type 1 are being used.
- For the **pessimistic bound analysis**, if sensing is conducted, then we assume it is on a channel in type 1 (higher SUs service rate), i.e., all available channels in type 2 are being used.

Throughout this section, we consider the optimistic bound performance, while formulating the global balance equations. With minor modifications of these global balance equations, we can also model the pessimistic bound performance.

Case 1: If $v < (C_1 - p_1)$, then all active SUs are using channels in C_1. Hence, the global balance equations are equations (1) and (2).

$$\begin{aligned}
&\pi_{v,p_1,p_2,1}[\lambda_s + v\mu_{s_1} + p_1\mu_{p_1} + p_2\mu_{p_2} + (C_1 - p_1)\lambda_{p_1} + \\
&(C_2 - p_2)\lambda_{p_2} + \Psi_{(p1+p_2,v)}(1 - p_f)] \\
&= \pi_{v-1,p_1,p_2,0}\left[\lambda_s\right]\mathbb{1}_{v\geq 1} + \pi_{v-1,p_1,p_2,1} \\
&\left[\lambda_s\right]\mathbb{1}_{v\geq 2} + \pi_{v+1,p_1,p_2,1}\left[(v+1)\mu_{s_1}\right] + \pi_{v,p_1+1,p_2,1} \\
&\left[(p_1+1)\mu_{p_1}\right]\mathbb{1}_{v\geq 1} + \pi_{v,p_1,p_2+1,1}\left[(p_2+1)\mu_{p_2}\right]\mathbb{1}_{v\geq 1} \\
&+ \pi_{v,p_1,p_2-1,1}\left[(C_2 - p_2 + 1)\lambda_{p_2}\right]\mathbb{1}_{v\geq 1} \\
&+ \frac{v}{C_1 - p_1 + 1}\pi_{v,p_1-1,p_2,0}\left[(C_1 - p_1 + 1)\lambda_{p_1}\right] \\
&+ \frac{C_1 - p_1 + 1 - v}{C_1 - p_1 + 1}\pi_{v,p_1-1,p_2,1}\left[(C_1 - p_1 + 1)\lambda_{p_1}\right]\mathbb{1}_{v\geq 1}.
\end{aligned} \tag{1}$$

Sensing is considered for all channels, with channels in C_1 given a higher priority when sensed by SUs. In equation (1), $\mathbb{1}_{v\geq x}$ is an indicator function which equals 1 if the condition, $v \geq x$, holds. Otherwise, it is 0. The LHS of equation (1), is the probability flux of leaving state $(v, p_1, p_2, 1)$ due to: an SU arrival with rate λ_s, an SU in C_1 finishes its transmission with rate $v\mu_{s_1}$, a PU in C_1 or C_2 finishing its transmission with rates $p_1\mu_{p_1}$ and $p_2\mu_{p_2}$, respectively, a PU arrived to C_1 or C_2 with rates $(C_1 - p_1)\lambda_{p_1}$ or $(C_2 - p_2)\lambda_{p_2}$, respectively, and end of sensing with rate of $\Psi_{(p1+p_2,v)}(1 - p_f)$.

The RHS of equation 1, is probability flux of entering state $(v, p_1, p_2, 1)$. This is due to: an SU arrived while the system in states $(v-1, p_1, p_2, 0)$ and $(v-1, p_1, p_2, 1)$. An SU finishing its transmission while the system in state $(v + 1, p_1, p_2, 1)$ with rate $(v + 1)\mu_{s_1}$, a PU completing service while the system is in state $(v, p_1 + 1, p_2, 1)$ or $(v, p_1, p_2 + 1, 1)$, with rates $(p_1 + 1)\mu_{p_1}$ or $(p_2 + 1)\mu_{p_2}$, respectively, a PU arriving to C_2 with rate $(C_2 - p_2 + 1)\lambda_{p_2}$, while the system in state in state $(v, p_1, p_2 - 1, 1)$, a PU arriving to C_1 while in state $(v, p_1 - 1, p_2, 0)$, and interrupting an SU which is using its channel, thus sensing by the SU is triggered with probability $\frac{v}{C_1-p_1+1}$, and a PU arriving to its channel which is not being used by an SU, with probability $\frac{C_1-p_1+1-v}{C_1-p_1+1}$, while the system is in state $(v, p_1 - 1, p_2, 1)$, given there has been sensing. In the rest of the paper, only the new transition states will be explained, due to space limitation.

In equation (2), the LHS is similar to that in equation (1), but there is no sensing. In the RHS, the second term to the last, the system transits from state $(v, p_1 - 1, p_2, 0)$ to state $(v, p_1, p_2, 0)$, due to a PU arrival to its channel in C_1 where no SU exits, with a probability of $\frac{C_1-p_1+1-v}{C_1-p_1+1}$, given there was no sensing.

Case 2: If $(C_1 - p_1) \leq v < (C_1 - p_1) + (C_2 - p_2)$, then the global balance equations are (3) and (4). In this case, if PU being served within C_1 finishes its transmission, SUs sensing is triggered. Also, these equations implicitly model the SUs preference to be served by C_1 channels rather than C_2 channels. Thus, when a PU in C_1 finishes its transmission, say at channel k, sensing is triggered, and then an SU moves to channel k.

$$\begin{aligned}
&\pi_{v,p_1,p_2,0}[\lambda_s + v\mu_{s_1} + p_1\mu_{p_1} + p_2\mu_{p_2} + (C_1 - p_1)\lambda_{p_1} + (C_2 \\
&- p_2)\lambda_{p_2}] = \pi_{v,p_1,p_2,1}\left[\Psi_{(p1+p_2,v)}(1 - p_f)\right]\mathbb{1}_{v\geq 1} \\
&+ \pi_{v+1,p_1,p_2,0}\left[(v+1)\mu_{s_1}\right] + \pi_{v,p_1+1,p_2,0}\left[(p_1+1)\mu_{p_1}\right] \\
&+ \pi_{v,p_1,p_2+1,0}\left[(p_2+1)\mu_{p_2}\right] \\
&+ \frac{C_1 - p_1 + 1 - v}{C_1 - p_1 + 1}\pi_{v,p_1-1,p_2,0}\left[(C_1 - p_1 + 1)\lambda_{p_1}\right] \\
&+ \pi_{v,p_1,p_2-1,0}\left[(C_2 - p_2 + 1)\lambda_{p_2}\right].
\end{aligned} \tag{2}$$

In equation (3) in the last term on the RHS, a PU arrives to its channel in C_2, where no SU is using it, with probability $\frac{C_2-p_2+1-(v-C_1-p_1)}{C_2-p_2+1}$, given sensing was not being conducted. Thus, system transits to state $(v, p_1, p_2, 0)$ (LHS). In equation (4), the system transits to state $(v, p_1, p_2, 1)$ in LHS, from different states, for example: from state $(v, p_1, p_2 - 1, 0)$ with a probability of $\frac{v-C_1-p_1}{C_2-p_2+1}$, when a PU arrives to C_2 and interrupts an SU that is using its channel. Thus, the PU arrival causes sensing to start. The same thing occurs in the second term to last, in state $(v, p_1, p_2-1, 1)$ with the same probability. However, in this case an SU which was already engaged in sensing will just continue to sense. However, in the last term a PU arrives to its channel in C_2, where no SU is using it, with probability $\frac{C_2-p_2+1-(v-C_1-p_1)}{C_2-p_2+1}$, given sensing was being conducted.

Case 3: If $v = (C_1 - p_1) + (C_2 - p_2)$, then the global balance equations are equations (5) and (6). In equation (5) the last term in RHS, shows sensing is triggered by a PU interruption of an SU which was served by the PU's channel, with probability $\frac{C_2-p_2}{C_2-p_2+1}$, given there is still one free channel in C_2. However, in equation (6), the last term on the RHS corresponds to a PU arriving to a channel where no SU was being served, with probability of $\frac{1}{C_2-p_2+1}$, and hence, sensing is not triggered.

Case 4: If $(C_1 - p_1) + (C_2 - p_2) < v < \beta$, then, equations (7) and (8) are the global balance equations. Recall that for the optimistic bound analysis, if sensing is conducted, then it is at a channel in C_2, i.e., all available channels in C_1 are being used.

$$\begin{aligned}
&\pi_{v,p_1,p_2,0}[\lambda_s + (C_1 - p_1)\mu_{s_1} + (v - C_2 + p_2)\mu_{s_2} + p_1\mu_{p_1} \\
&+ p_2\mu_{p_2} + (C_1 - p_1)\lambda_{p_1} + (C_2 - p_2)\lambda_{p_2}] \\
&= \pi_{v,p_1,p_2,1}\left[\Psi_{(p1+p_2,v)}(1 - p_f)\right]\mathbb{1}_{v\geq 1} \\
&+ \pi_{v+1,p_1,p_2,0}\left[(v + 1 - C_1 + p_1)\mu_{s_2}\right] + \pi_{v,p_1,p_2+1,0} \\
&\left[(p_2+1)\mu_{p_2}\right] + \frac{C_2 - p_2 + 1 - (v - C_1 + p_1)}{C_2 - p_2 + 1} \\
&\pi_{v,p_1,p_2-1,0}\left[(C_2 - p_2 + 1)\lambda_{p_2}\right].
\end{aligned} \tag{3}$$

$$\pi_{v,p_1,p_2,1}[\lambda_s + (C_1 - p_1)\mu_{s_1} + (v - C_1 + p_1 - 1 \mathbb{1}_{(v-C_1+p_1)\geq 1})\mu_{s_2} + p_1\mu_{p_1} + p_2\mu_{p_2} + (C_1 - p_1)\lambda_{p_1} + (C_2 - p_2)\lambda_{p_2} + \Psi_{(p1+p_2,v)}(1 - p_f)] = \pi_{v-1,p_1,p_2,0}\,[\lambda_s]\,\mathbb{1}_{v\geq 1} + \pi_{v-1,p_1,p_2,1}\,[\lambda_s]\,\mathbb{1}_{v\geq 2} + \pi_{v+1,p_1,p_2,0}\,[(C_1 - p_1)\mu_{s_1}] + \pi_{v+1,p_1,p_2,1}\,[(C_1 - p_1)\mu_{s_1}] + \pi_{v+1,p_1,p_2,1}\,[(v + 1 - C_1 + p_1 - 1)\mu_{s_2}] + \pi_{v,p_1+1,p_2,0}[(p_1 + 1)\mu_{p_1}] + \pi_{v,p_1+1,p_2,1}\,[(p_1 + 1)\mu_{p_1}]\,\mathbb{1}_{v\geq 1} + \pi_{v,p_1,p_2+1,1}\,[(p_2 + 1)\mu_{p_2}]\,\mathbb{1}_{v\geq 1} + \pi_{v,p_1-1,p_2,1}\,[(C_1 - p_1 + 1)\lambda_{p_1}]\,\mathbb{1}_{v\geq 1} + \pi_{v,p_1-1,p_2,0}\,[(C_1 - p_1 + 1)\lambda_{p_1}] + \frac{v - C_1 + p_1}{C_2 - p_2 + 1}\pi_{v,p_1,p_2-1,0}\,[(C_2 - p_2 + 1)\lambda_{p_2}] + \frac{v - C_1 + p_1}{C_2 - p_2 + 1}\pi_{v,p_1,p_2-1,1}\,[(C_2 - p_2 + 1)\lambda_{p_2}]\,\mathbb{1}_{v\geq 1} + \frac{C_2 - p_2 + 1 - (v - C_1 + p_1)}{C_2 - p_2 + 1}\pi_{v,p_1,p_2-1,1}[(C_2 - p_2 + 1)\lambda_{p_2}]\mathbb{1}_{v\geq 1}. \tag{4}$$

$$\pi_{v,p_1,p_2,1}[\lambda_s + (C_1 - p_1)\mu_{s_1} + (C_2 - p_2 - 1)\mu_{s_2}\mathbb{1}_{C_2-p_2\neq 0} + p_1\mu_{p_1} + p_2\mu_{p_2} + (C_1 - p_1)\lambda_{p_1} + (C_2 - p_2)\lambda_{p_2} + \Psi_{(p1+p_2,v)}(1 - p_f)] = \pi_{v-1,p_1,p_2,0}\,[\lambda_s]\,\mathbb{1}_{v\geq 1} + \pi_{v-1,p_1,p_2,1}\,[\lambda_s]\,\mathbb{1}_{v\geq 2} + \pi_{v+1,p_1,p_2,0}\,[(C_1 - p_1)\mu_{s_1}] + \pi_{v+1,p_1,p_2,0}\,[(C_2 - p_2)\mu_{s_2}] + \pi_{v+1,p_1,p_2,1}\,[(C_1 - p_1)\mu_{s_1}] + \pi_{v+1,p_1,p_2,1}\,\left[(C_2 - p_2 - 1\mathbb{1}_{(C_2-p_2)\geq 1})\mu_{s_2}\right] + \pi_{v,p_1+1,p_2,0}\,[(p_1 + 1)\mu_{p_1}] + \pi_{v,p_1,p_2+1,0}\,[(p_2 + 1)\mu_{p_2}] + \pi_{v,p_1+1,p_2,1}\,[(p_1 + 1)\mu_{p_1}]\,\mathbb{1}_{v\geq 1} + \pi_{v,p_1,p_2+1,1}\,[(p_2 + 1)\mu_{p_2}]\,\mathbb{1}_{v\geq 1} + \pi_{v,p_1-1,p_2,0}\,[(C_1 - p_1 + 1)\lambda_{p_1}] + \pi_{v,p_1-1,p_2,1}\,[(C_1 - p_1 + 1)\lambda_{p_1}]\,\mathbb{1}_{v\geq 1} + \pi_{v,p_1,p_2-1,1}\,[(C_2 - p_2 + 1)\lambda_{p_2}]\,\mathbb{1}_{v\geq 1} + \frac{C_2 - p_2}{C_2 - p_2 + 1}\pi_{v,p_1,p_2-1,0}\,[(C_2 - p_2 + 1)\lambda_{p_2}]\,. \tag{5}$$

$$\pi_{v,p_1,p_2,0}[\lambda_s + (C_1 - p_1)\mu_{s_1} + (C_2 - p_2)\mu_{s_2} + p_1\mu_{p_1} + p_2\mu_{p_2} + (C_1 - p_1)\lambda_{p_1} + (C_2 - p_2)\lambda_{p_2}] = \pi_{v,p_1,p_2,1}\,\left[\Psi_{(p1+p_2,v)}(1 - p_f)\right]\,\mathbb{1}_{v\geq 1} + \frac{1}{C_2 - p_2 + 1}\pi_{v,p_1,p_2-1,0}\,[(C_2 - p_2 + 1)\lambda_{p_2}]\,. \tag{6}$$

In equations (7) and (9), the last term in RHS corresponds to a PU arriving to a channel where an SU is sensing it, which occurs with probability of $\frac{1}{C_2-p_2+1}$, hence sensing is terminated. However, in Equation (8) the last term in RHS, shows that the sensing has not been terminated , since the PU arrives to a channel where sensing is not being conducted, with probability of $\frac{C_2-p_2}{C_2-p_2+1}$. However, the PU arrival causes an SU interruption, where the SU goes back to the VQ, and waits for a channel to become available. Recall that we assume sensing is always conducted at a channel in type 2.

Case 5: In this case $v = \beta$. As a result, equations (9) and (10) are the global balance equations.

$$\pi_{v,p_1,p_2,0}[\lambda_s + (C_1 - p_1)\mu_{s_1} + (C_2 - p_2)\mu_{s_2} + p_1\mu_{p_1} + p_2\mu_{p_2} + (C_1 - p_1)\lambda_{p_1} + (C_2 - p_2)\lambda_{p_2}] = \pi_{v,p_1,p_2,1}\,\left[\Psi_{(p1+p_2,v)}(1 - p_f)\right]\,\mathbb{1}_{v\geq 1} + \pi_{v-1,p_1,p_2,0}\,[\lambda_s] + \pi_{v,p_1-1,p_2,0}\,[(C_1 - p_1 + 1)\lambda_{p_1}] + \pi_{v,p_1,p_2-1,0}\,[(C_2 - p_2 + 1)\lambda_{p_2}] + \frac{1}{C_2 - p_2 + 1}\pi_{v,p_1,p_2-1,1}\,[(C_2 - p_2 + 1)\lambda_{p_2}]\,\mathbb{1}_{v\geq 1}. \tag{7}$$

$$\pi_{v,p_1,p_2,1}[\lambda_s + (C_1 - p_1)\mu_{s_1} + (C_2 - p_2 - 1 \mathbb{1}_{(C_2-p_2)\neq 0})\mu_{s_2} + p_1\mu_{p_1} + p_2\mu_{p_2} + (C_1 - p_1)\lambda_{p_1} + (C_2 - p_2)\lambda_{p_2} + \Psi_{(p1+p_2,v)}(1 - p_f)] = \pi_{v-1,p_1,p_2,1}\,[\lambda_s]\,\mathbb{1}_{v\geq 2} + \pi_{v+1,p_1,p_2,1}\,[(C_1 - p_1)\mu_{s_1}] + \pi_{v+1,p_1,p_2,1}\,\left[(C_2 - p_2 - 1\mathbb{1}_{(C_2-p_2)\geq 1}))\mu_{s_2}\right] + \pi_{v+1,p_1,p_2,0}\,[(C_1 - p_1)\mu_{s_1}] + \pi_{v+1,p_1,p_2,0}\,[(C_2 - p_2)\mu_{s_2}] + \pi_{v,p_1+1,p_2,1}\,[(p_1 + 1)\mu_{p_1}]\,\mathbb{1}_{v\geq 1} + \pi_{v,p_1,p_2+1,1}\,[(p_2 + 1)\mu_{p_2}]\,\mathbb{1}_{v\geq 1} + \pi_{v,p_1+1,p_2,0}\,[(p_1 + 1)\mu_{p_1}] + \pi_{v,p_1,p_2+1,0}\,[(p_2 + 1)\mu_{p_2}] + \pi_{v,p_1-1,p_2,1}\,[(C_1 - p_1 + 1)\lambda_{p_1}]\,\mathbb{1}_{v\geq 1} + \frac{C_2 - p_2}{C_2 - p_2 + 1}\pi_{v,p_1,p_2-1,1}\,[(C_2 - p_2 + 1)\lambda_{p_2}]\,\mathbb{1}_{v\geq 1}. \tag{8}$$

$$\pi_{\beta,p_1,p_2,0}[(C_1 - p_1)\mu_{s_1} + (C_2 - p_2)\mu_{s_2} + p_1\mu_{p_1} + p_2\mu_{p_2} + (C_1 - p_1)\lambda_{p_1} + (C_2 - p_2)\lambda_{p_2}] = \pi_{\beta-1,p_1,p_2,0}\,[\lambda_s]\,\mathbb{1}_{\beta\geq 1} + \pi_{\beta,p_1,p_2,1}\,\left[\Psi_{(p1+p_2,v)}(1 - p_f)\right]\,\mathbb{1}_{\beta\geq 1} + \pi_{\beta,p_1-1,p_2,0}\,[(C_1 - p_1 + 1)\lambda_{p_1}] + \pi_{\beta,p_1,p_2-1,0}\,[(C_2 - p_2 + 1)\lambda_{p_2}] + \frac{1}{C_2 - p_2 + 1}\pi_{\beta,p_1,p_2-1,1}\,[(C_2 - p_2 + 1)\lambda_{p_2}]\,\mathbb{1}_{\beta\geq 1}. \tag{9}$$

$$\pi_{\beta,p_1,p_2,1}[(C_1 - p_1)\mu_{s_1} + (C_2 - p_2 - 1\mathbb{1}_{(C_2-p_2)\neq 0})\mu_{s_2} + p_1\mu_{p_1} + p_2\mu_{p_2} + (C_1 - p_1)\lambda_{p_1} + (C_2 - p_2)\lambda_{p_2} + \Psi_{(p1+p_2,v)}(1 - p_f)] = \pi_{\beta-1,p_1,p_2,1}\,[\lambda_s]\,\mathbb{1}_{\beta\geq 2} + \pi_{\beta,p_1+1,p_2,1}\,[(p_1 + 1)\mu_{p_1}]\,\mathbb{1}_{\beta\geq 1} + \pi_{\beta,p_1,p_2+1,1}\,[(p_2 + 1)\mu_{p_2}]\,\mathbb{1}_{\beta\geq 1} + \pi_{\beta,p_1+1,p_2,0}\,[(p_1 + 1)\mu_{p_1}] + \pi_{\beta,p_1,p_2+1,0}\,[(p_2 + 1)\mu_{p_2}] + \pi_{\beta,p_1-1,p_2,1}\,[(C_1 - p_1 + 1)\lambda_{p_1}]\,\mathbb{1}_{\beta\geq 1} + \frac{C_2 - p_2}{C_2 - p_2 + 1}\pi_{\beta,p_1,p_2-1,1}\,[(C_2 - p_2 + 1)\lambda_{p_2}]\,\mathbb{1}_{\beta\geq 1}. \tag{10}$$

IV. Performance Metrics

In this section, we introduce several performance metrics which can be used to evaluate CRN performance. These include the probabilities of admission and blocking of SUs, average number of SUs in the system during the network operation, and average waiting time for SUs in the system until completing service. We solved the steady state probability distribution, $\overrightarrow{\pi}$, by solving the equation $\overrightarrow{\pi}Q = 0$, where Q is the transition rate matrix that can be constructed using the global balance equations (1)−(10).

However, the number of linearly independent global balance equation is $(m-1)$. Therefore, use the fact that the summation of all probabilities in the steady state distribution equals 1. As a results, we have m linearly independent solvable equations.

Let us introduce the following definitions:

Definition IV.1 *Probability of blocking of SUs (p_b): It is the probability that a new SU request for transmission is blocked due to the lack of space in the VQ.*

Definition IV.2 *Probability of admission for SUs (p_a): It is the probability that a new SU request for transmission is admitted.*

Definition IV.3 *The average number of SUs in the system ($\overline{L}$), which includes those being served by channels of types 1 and 2, and also those waiting for a channel to become available.*

Definition IV.4 *Average waiting time ($\overline{W}$) of SUs, which is measured from the instant of arrival, until finishing its transmission.*

The following equations are used to evaluate the performance metrics of our proposed model.

1) The probability of blocking for SUs (p_b) is given by equation (11).
$$p_b = \sum_{p_1=0}^{C_1} \sum_{p_2=0}^{C_2} \sum_{s=0}^{1} \Bigg[\sum_{\substack{\{v=\beta\},\\ (v,p_1,p_2,s)\in\zeta}} \pi_{v,p_1,p_2,s} \Bigg]. \qquad (11)$$
2) The probability of SU admission, (p_a), equals to $1 - p_b$.
3) The average number of SUs in the system ($\overline{L}$), is given by equation (12).
$$\overline{L} = \sum_{p_1=0}^{C_1} \sum_{p_2=0}^{C_2} \sum_{s=0}^{1} \Bigg[\sum_{\substack{v=1,\\ (v,p_1,p_2,s)\in\zeta}}^{\beta} v \times \pi_{v,p_1,p_2,s} \Bigg]. \qquad (12)$$
4) To find the average waiting time $\overline{W}$, we appeal to Little's Theorem, where $\overline{L}$ is given by equation (12), and $\overline{W}$ is expressed as $\overline{W} = \frac{\overline{L}}{p_a \times \lambda_s}$.

V. Baseline Model

In this section, we introduce and model another system. This is a system similar to our proposed model, but with no channel switching to type 1 channels (if available) by SUs which are being served in type 2 channels, unless there are no longer available channels on type 2. We developed this system and use it as a baseline model to establish the advantages of our proposed approach. For example, if an SU arrives and selects a channel, say from set C_2, the SU keeps using this channel, until finishing its transmission, as long as this channel is available. However, if the SU is interrupted, and sense there are no available channels on type 2 to use it, and there is an available channel on type 1, therefore the SU switches to this channel. Otherwise, when no channel is available in both types 1 and 2, the SU is buffered in the VQ, until a channel becomes available. Due to space limitation in this paper, we did not write the global balance equations for the baseline model. The performance metrics for the baseline model are similar to our model, in Section IV. Similarly, those equations with minor modifications are used to evaluate the baseline model performance metrics, and due to space limitation in this report, we did not rewrite these equations.

VI. Numerical Results

This section presents the numerical results for SUs average waiting time, with respect to SUs arrival rate to the system, λ_s. Also, we study the effect of SUs sensing rate on the system performance.

The sensing rate is dependent on both the number of unused channels, and on the number of SUs performing the sensing process. It was proven in [14] that the expected time to detect an unused channel is inversely proportional to the number of unused channels, which means that the sensing rate is proportional to this number. Moreover, if the total number of channels very large, and is evenly divided among the SUs sensing for available channels (out-of-band sensing), then the rate of detecting an empty channel is the sum of the individual SUs sensing rates. We therefore express the sensing rate, as a function of p_1, p_2, and v, $\Psi_{(p_1+p_2,v)}$, as shown in equation (13), where η is the sensing rate when there is only a single SU sensing, and there is only one available channel.
$$\Psi_{(p_1+p_2,v)} = \eta \hat{N} \hat{I}. \qquad (13)$$

$\hat{N}$ is the number of SUs in the system which are not being served by channels (waiting/interrupted) or want to improve their performance by switching to a channel in type 1, and therefore conduct out-of-band sensing. $\hat{I}$ is number of idle channels in type 1 and 2 channels which are not being used by SUs or PUs. Based on the global balance equations (1)$-$(10), if $v \leq (C_1 - p_1)$, $\hat{N} = 0$ (No SUs are interested in sensing, since all current SUs are being served by type 1 channels). If $(C_1-p_1) < v \leq (C_1-p_1+C_2-p_2)$, then $\hat{N} = v-(C_1-p_1)$. Otherwise, $\hat{N} = v - (C_1 - p_1 + C_2 - p_2)$. If $v \leq (C_1 - p_1 + C_2 - p_2)$, then $\hat{I} = (C_1 - p_1 + C_2 - p_2) - v$. Otherwise, $\hat{I} = 1$. According to equation (13) the sensing rate increases (the sensing time decreases) when more SUs are active and sensing the channels. The sensing rate decreases (the sensing time increases) when more PUs are active, and therefore there are fewer available channels, and it takes longer to search for and sense those channels.

In order to evaluate our proposed model, we consider two different scenarios as follows, it is worth mentioning that in both scenarios, SUs service rates in type 1 and 2 channels are different.

- **Scenario** 1 parameters, $\mu_{s_1} = 60$, $\mu_{s_2} = 15$, $\lambda_{p_1} = \lambda_{p_2} = 5$, $\mu_{p_1} = \mu_{p_2} = 10$, $\eta = 250$, $C_1 = C_2 = 4$, $p_f = 0.09$, and $\beta = 40$.
- **Scenario** 2 parameters, $\mu_{s_1} = 80$, $\mu_{s_2} = 20$, $\lambda_{p_1} = 5$, $\lambda_{p_2} = 25$, $\mu_{p_1} = 15$, $\mu_{p_2} = 80$, $\eta = 380$, $C_1 = C_2 = 3$, $p_f = 0.05$, and $\beta = 50$. This scenario, is different from the first one, where PUs arrival and service rates are not equal in both channel types. Also, PUs service time in type 1 channels is greater than those in type 2, to capture the heterogeneity nature, such as in TV channels and cellular phones channels.

A. Average waiting time of SUs:

We show how the SUs' average waiting time, $\overline{W}$, changes with respect to SUs arrival rate, λ_s. **It is worth mentioning the probability of SUs admission, p_a, in our model and the baseline model for all numerical results in this section**

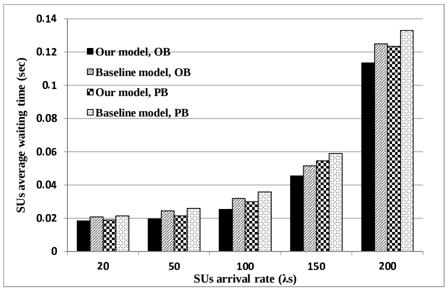

Fig. 1. Scenario 1, SUs $\overline{W}$ time with respect to their arrival rate, λ_s.

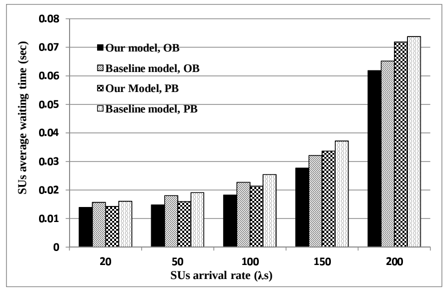

Fig. 2. Scenario 2, SUs $\overline{W}$ time with respect to their arrival rate, λ_s.

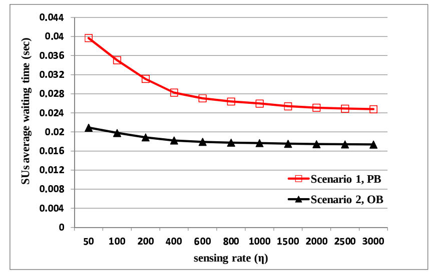

Fig. 3. SUs $\overline{W}$ time, with respect to their sensing rate, η, for scenarios 1 and 2 for PB and OB, respectively, where λ_s is fixed and set to 100.

is almost 1. In our results, β is set to a value, e.g., in scenario 2 $\beta = 50$, such that p_a is almost 1. We have varied β size up to 100 in both Scenarios studies, however, this do not change the numerical results, e.g., $\overline{W}$. Therefore, we approximate the infinite number of SUs case in our results. Figure 1 which corresponds to scenario 1, shows that $\overline{W}$ increases by increasing SUs arrival rate, λ_s. Our model outperforms the baseline model in both the Optimistic Bound (OB) and Pessimistic Bound (PB) analysis, because our model reduces $\overline{W}$ for SUs in the system. For example, for the OB analysis, and when $\lambda_s = 20$, 100, and 150, our model reduces $\overline{W}$ by up to 12.44%, 20.68%, and 11.99%, respectively, with respect to the baseline model. One observation, when $\lambda_s = 100$, $\overline{W}$ reduction is higher than when $\lambda_s = 150$. Therefore, our model $\overline{W}$ reduction percentage over the baseline model reaches its maximum value, when λ_s increased to some value, and then this percentage decreases.

Also, Figure 2, which corresponds to scenario 2 system parameters, shows that $\overline{W}$ increases by increasing λ_s. This figure, shows although the PUs behavior is different between the two types of channels, our model outperforms the baseline model in the OB and PB analysis. For example, for the PB analysis, and when $\lambda_s = 50$, our model reduces $\overline{W}$ by up to 16.23% with respect to the baseline model. Please notice that sensing rate, equation (13), is higher than the SUs and PUs arrival and service rates in these cases studies.

B. Sensing Rate:

We consider the Pessimistic and the Optimistic bounds analysis for scenarios 1 and 2, respectively, in our model to study the effect of sensing rate, η, on SUs performance. The system parameters correspond to scenarios 1 and 2 parameters, except that in both scenarios λ_s is fixed and is set to 100, while the sensing rate is varied on the X axis. Figure 3 shows that the SUs' average waiting time, $\overline{W}$, decreases by increasing η. The smallest value for sensing rate in this figure is 50, i.e., an average sensing time of 20 ms, which is about the sensing time using feature detection [13]. However, for the energy detection method, the sensing time is ≤ 1 ms [13], or $\eta \geq 1000$. Clearly, when the energy detection method is used instead of feature detection, the SUs performance is better and $\overline{W}$ decreases. This figure also shows that if η is increased beyond 1000, its effect on SUs performance is insignificant for both cases studies scenarios, e.g., in scenario 1 for the PB, when η is increased from 50 to 1000, $\overline{W}$ is reduced by up to 34.56%, however, when η is increased from 1000 to 2000, $\overline{W}$ is only reduced by up to 3.42%. Since our model discards SU arrivals occurring while channels are being sensed, $\overline{W}$ in Figure 3 is underestimated.

VII. Conclusions

In this paper we proposed a model for heterogeneous channel access in Cognitive Radio Networks (CRNs). In this model, there are two types of licensed channels, where one type has a larger bandwidth. SUs may use the first type if it is available, or if it becomes available. We also model the SUs' sensing process and study its effect on performance, such that sensing rate is dependent on both the number of unused channels, and on the number of SUs performing the sensing process. We used a mixed queuing network model to model the CRN system, and developed the global balance equations for a $CTMC$. We derived SUs' performance metrics, such as SUs admission and blocking probabilities, and their average waiting time in the system. We compare our proposed system to a baseline model, which is the same as our proposed model, except that SUs in type 2 channel can not improve their throughput by switching to channels in type 1, if available, unless they are interrupted at their current type 2 channels. Numerical results show that our proposed model outperforms the baseline model. We also found that if sensing time is very small ($\leq$1 ms), its effect on SUs performance is insignificant.

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